Uber Pulley Problem

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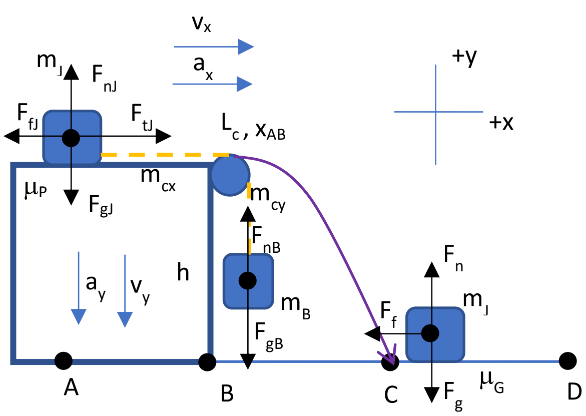
Description:

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper, he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his (net) speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

Diagram:



Assumptions:

$F_{tB} = F_{tJ}$ $\quad a_x = -a_y$

Givens:

$m_J = 74\ kg$ $\quad \Delta x_{AB} = 12\ m$

$m_B = 169\ kg$ $\quad \Delta x_{BD} = 74\ m$

$m_C = 70\ kg$ $\quad$ EQ 1: $\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$

$L_C = 12\ m$ $\quad$ EQ 2: $v_f = a\Delta t + v_i$

$h = 21\ m$ $\quad$ EQ 3: $x_f = \frac{1}{2}a\Delta t^2 + v_i\Delta t + x_i$

$\mu_P = 0.13$ $\quad$ EQ 4: $v_f^2 = v_i^2 + 2a\Delta x$

Stage AB:

The goal of this stage is to find the velocity of Jerry and the Jumper at point B. This can be done by summing all of the forces acting on Jerry in the x and y direction. The mass on either side of the pulley is constantly changing because, as the chain moves, more mass gets removed from the platform. This is modeled by the equations $m_{cy}[x]$ and $m_{cy}[x]$, which output the mass of chain on the platform and off of the ledge in relation on Jerry's x position. These components can then be used to derive an equation for acceleration in the x-direction in relation on Jerry's x position ($a_x[x]$). Through calculus, the velocity can then be found using the acceleration equation.

Equation for total mass in the x-direction:

$$m_{cx}[x] = m_c - \frac{x}{L_c} \times m_c$$

$$m_x[x] = m_J + m_{cx}[x]$$

$$m_x[x] = m_j + \left(m_c - \frac{x}{L_c} \times m_c\right)$$

$$m_x[x] = 74 + \left(70 - \frac{x}{12} \times 70\right)$$

$$\underline{m_x[x] = 144 - 5.8333x}$$

Equation for total mass in the y-direction:

$$m_{cy}[x] = \frac{x}{L_c} \times m_c$$

$$m_y[x] = m_B + m_{cy}[x]$$

$$m_y[x] = m_B + \frac{x}{L_c} \times m_c$$

$$m_y[x] = 169 + \frac{x}{12} \times 70$$

$$\underline{m_y[x] = 169 + 5.8333x}$$

Sum of forces in the x-direction:

$$\sum F_x: F_{tJ} - F_{fJ} = m_x[x]a_x$$

$$F_{tJ} - um_J g = m_x[x]a_x$$

$$F_{tJ} = m_x[x]a_x + um_J g$$

Sum of forces in the y -direction:

$$\sum F_y: F_{tB} - F_{gB} = m_y[x]a_y$$

$$F_{tB} - m_y[x]g = m_y[x] * -a_x$$

$$F_{tB} = -m_y[x]a_x + m_y[x]g$$

Derive equation for $a_x[x]$ by setting $F_{tB} = F_{tJ}$:

$$F_{tB} = F_{tJ}$$

$$-m_y[x]a_x + m_y[x]g = m_x[x]a_x + um_J g$$

$$-m_y[x]a_x - m_x[x]a_x = um_J g - m_y[x]g$$

$$a_x[x] = \frac{m_y[x]g - um_J g}{m_y[x] + m_x[x]}$$

$$a_x[x] = \frac{m_y[x] * 9.8 - (0.13)(74)(9.8)}{m_y[x] + m_x[x]}$$

$$a_x[x] = \frac{(169 + 5.8333x)9.8 - (0.13)(74)(9.8)}{(169 + 5.8333x) + (144 - 5.8333x)}$$

$$a_x[x] = \frac{1656.2 + 57.1663x - 94.276}{313}$$

$$\underline{a_x[x] = 0.1826x + 4.99017}$$

Finding $v_x[x]$ by using definition of acceleration:

$$a \equiv \frac{dv}{dt}$$
$$a = \frac{dx}{dx} * \frac{dv}{dt}$$
$$a = v * \frac{dv}{dx}$$
$$a_x[x] * dx = v * dv$$
$$\int_{x_0}^{x} a_x[x]dx = \int_{v_0}^{v} (v)dv$$
$$\int_{0}^{x} (0.1826x + 4.99017)dx = \int_{0}^{v} (v)dv, solver$$
$$0.091321x^2 + 4.99017 = \frac{v^2}{2} - \frac{0^2}{2}$$
$$0.1826x^2 + 9.98035 = v^2$$
$$\underline{v[x] = \sqrt{0.1826x^2 + 9.98035}}$$
$$v[12] = \sqrt{0.1826(12)^2 + 9.98035}, solver$$
$$\underline{v[12] = v_{Bx} = 12.086 \frac{m}{s}}$$

Stage BC:

Using kinematics, this stage finds Jerry's velocity as he reaches the ground and the distance he travels form B to C. Jerry loses 25% of his net velocity as he hits the ground. These values are used in stage CD to find the coefficient of friction.

Y-Direction:

$$EQ\ 3: y_f = \frac{1}{2}(a_y)t^2 + (v_{By})t + y_i$$
$$y[t] = \frac{1}{2}(-9.8)t^2 + (0)t + 21$$

Time from point B to C:

$$-4.9t^2 + 21 = 0$$
$$t^2 = \frac{-21}{-4.9}$$
$$t = \sqrt{4.29}$$
$$\underline{t_{BC} = 2.07\ sec}, \cancel{t_{BC} = -2.07\ sec}$$

X-Direction:

$$EQ\ 3: x_f = \frac{1}{2}(a_x)t^2 + (v_{Bx})t + x_i$$
$$x_f = \frac{1}{2}(0)t^2 + (12.086)t + 0$$
$$x[t] = (12.086)t$$
$$x[2.07] = (12.086)(2.07)$$
$$\underline{x_{BC} = 25.0174\ m}$$

Velocity at point C in the x-direction:

$$EQ1: x_{BC} = \frac{1}{2}(v_{Cx} + v_{Bx})t_{BC}$$
$$25.0174 = \frac{1}{2}(v_{Cx} + 12.086)2.07$$
$$\frac{25.0174 * 2}{2.07} = v_{Cx} + 12.086$$
$$24.171 - 12.086 = v_{Cx}$$
$$\underline{v_{Cx} = 12.0857 \frac{m}{s}}$$

Velocity at point C in y-direction:

$$EQ2: v_{Cy} = g(t_{BC}) + v_{By}$$
$$v_{Cy} = -9.8(2.07) + 0$$
$$\underline{v_{Cy} = -20.286 \frac{m}{s}}$$

Net velocity:

$$v_{Cnet} = \sqrt{v_{Cx}^2 + v_{Cy}^2}$$
$$v_{Cnet} = \sqrt{12.09^2 + -20.286^2}$$
$$v_{Cnet} = \sqrt{146.168 + 411.522}$$
$$v_{Cnet} = \sqrt{557.69}$$
$$\underline{v_{Cnet} = 23.6133 \frac{m}{s}}$$

75% Velocity at C:

$$v_c = v_{Cnet} * 0.75$$
$$v_C = 23.6133 * 0.75$$
$$\underline{v_C = 17.71 \frac{m}{s}}$$

Stage CD:

This stage finds the coefficient of friction on the ground by using the Jerry's velocity at point C and the distance he travels from B to C. First, the distance from point C to D is found. Then, using that along with Jerry's velocity at C, his acceleration is calculated. A second sum of forces in the x-direction is then used to find the coefficient of friction using his acceleration.

Distance traveled from point C to D:

$$x_{CD} = x_{BD} - x_{BC}$$
$$x_{CD} = 74 - 25.0174$$
$$\underline{x_{CD} = 48.9826\ m}$$

Acceleration:

$$EQ\ 4: v_D^2 = v_C^2 + 2ax_{CD}$$
$$0 = 17.71^2 + 2 * a * 48.9826$$
$$-313.642 = 97.9652a$$
$$\underline{a = -3.202 \frac{m}{s^2}}$$

Coefficient of Friction:

$$\sum F_x: -F_f = m_J * a$$
$$-\mu_G * m_J * g = m_J * a$$
$$-\mu_G * 9.8 = -3.202$$
$$\boxed{\mu_G = 0.3267}$$