Easing Traffic

The Lincoln Tunnel Authority is about to complete a tunnel under the city centre. Owing to financial cutbacks the tunnel is only one lane in either direction. The traffic Manager has realised that there will be holdups at both ends of the tunnel during the morning and evening rush hours. Bearing in mind aspects of safety and the desire to produce the maximum flow of traffic at peak times, he wishes to put up signs indicating a maximum speed and the distance to be maintained between vehicles.

What recommendations would you make to the Traffic Manager on this matter?

Givens

In[∘]:=

```
vel = {30, 50, 70}
think = {30, 50, 70}
brake = {45, 125, 245}
dist = {75, 175, 315}
table1 = Transpose[{vel, dist}]
```

Out[∘]= {30, 50, 70}

Out[∘]= {30, 50, 70}

Out[∘]= {45, 125, 245}

Out[∘]= {75, 175, 315}

Out[∘]= {{30, 75}, {50, 175}, {70, 315}}

In[∘]:=

```
Text[Grid[Prepend[table1, {"vel", "dist"}],
  Alignment → Center, Dividers → {Center}, Spacings → {1, 1}]]
```

Out[∘]=

vel	dist
30	75
50	175
70	315

In[∘]:=

```
lp1 = ListPlot[table1, AxesLabel → {"vel", "dist"},
            PlotLabel → Style["Easing Traffic", Blue, 16],
            PlotStyle → {Red, PointSize[0.015]},
    PlotRange → {{0, 100}, {0, 350}},
            AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

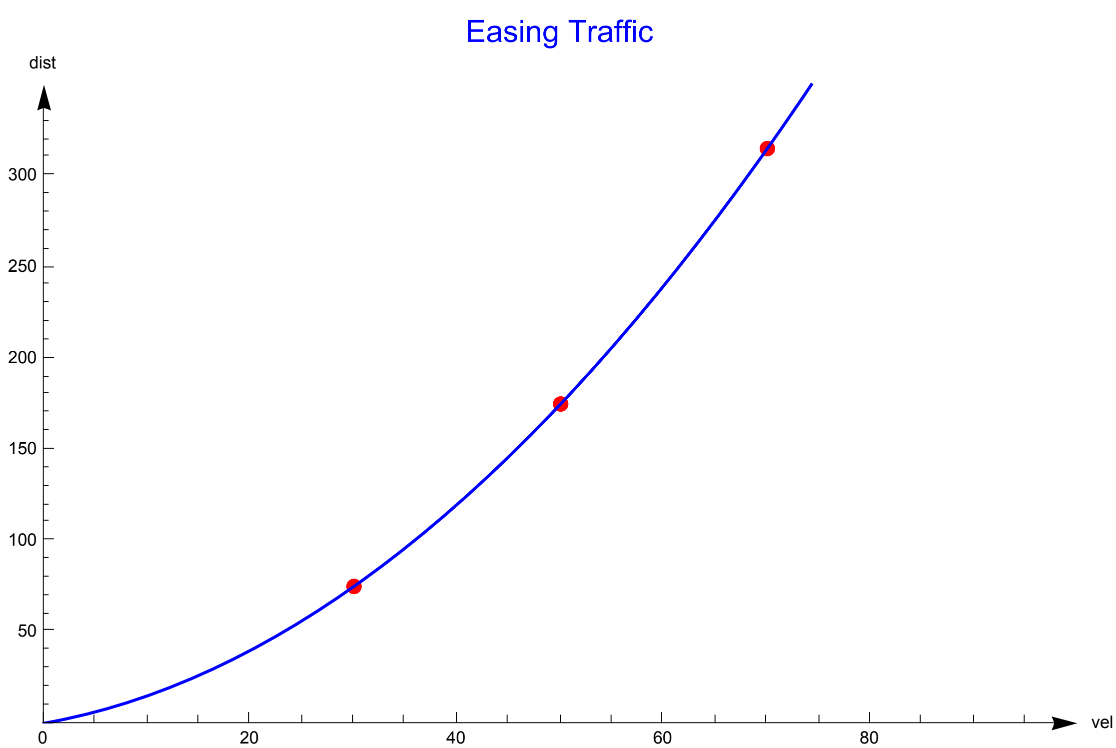
In[∘]:=
```
Fit[table1, {x, x^2}, x]
```
Out[∘]= $1.\,x + 0.05\,x^2$

In[∘]:=
```
dist1[x_] = 0.9999999999999967` x + 0.05000000000000002` x^2
```
Out[∘]= $1.\,x + 0.05\,x^2$

In[∘]:=
```
dist1[30]
```
Out[∘]= 75.

In[∘]:=
```
plot1 = Plot[dist1[x], {x, 0, 100}, AxesLabel → {"Velocity", "dist"},
                PlotLabel → Style["Easing Traffic", Blue, 16],
                PlotStyle → {Blue, PointSize[0.015]},
    PlotRange → {{0, 100}, {0, 350}},
                AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[∘]:=
```
Show[lp1, plot1]
```
Out[∘]=



In[∘]:=
```
vel2 = {5, 10, 20, 30, 40, 50, 60, 70}
```
Out[∘]= {5, 10, 20, 30, 40, 50, 60, 70}

In[∘]:=
```
dist2 = dist1[vel2]
```
Out[∘]= {6.25, 15., 40., 75., 120., 175., 240., 315.}

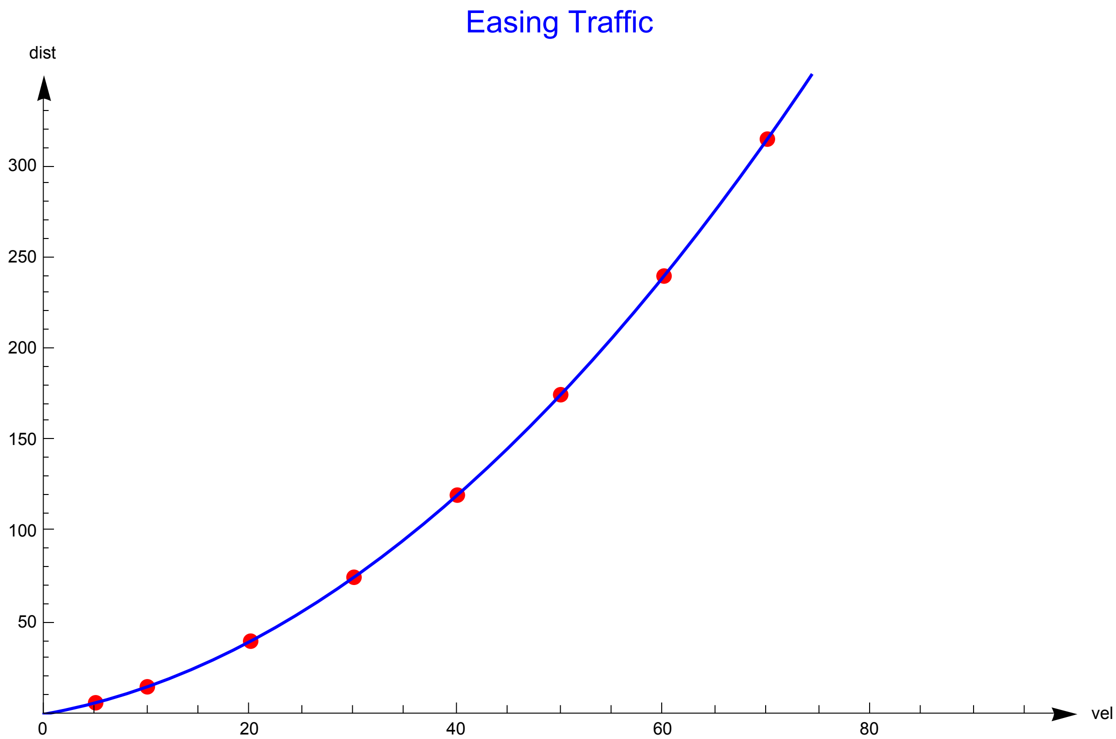
In[∘]:=
```
table2 = Transpose[{vel2, dist2}]
```

Out[∘]=
```
{{5, 6.25}, {10, 15.}, {20, 40.}, {30, 75.},
 {40, 120.}, {50, 175.}, {60, 240.}, {70, 315.}}
```

In[∘]:=
```
lp2 =  ListPlot[table2, AxesLabel → {"vel", "dist"},
                 PlotLabel → Style["Easing Traffic", Blue, 16],
                 PlotStyle → {Red, PointSize[0.015]},
    PlotRange → {{0, 100}, {0, 350}},
                 AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[∘]:=
```
Show[lp2, plot1]
```

Out[∘]=



I now have an equation relating the overall stopping distance to travel speed: dist[v] = 1* v + $0.05 * v^2$, dist (feet) and v (mph).

Model 1

In[∘]:=
```
ncarsf[vel2_, dist2_] := (88 * vel2) / dist2
```

In[∘]:=
```
ncarsf[5, 6.25]
```

Out[∘]=
```
70.4
```

In[∘]:=
```
ncars1 = Floor[ncarsf[vel2, dist2]]
```

Out[∘]=
```
{70, 58, 44, 35, 29, 25, 22, 19}
```

In[∘]:=
```
table2 = Transpose[{vel2, ncars1}]
```
Out[∘]=
```
{{5, 70}, {10, 58}, {20, 44}, {30, 35}, {40, 29}, {50, 25}, {60, 22}, {70, 19}}
```

In[∘]:=
```
Text[Grid[Prepend[table2, {"vel2", "ncars1"}],
  Alignment → Center, Dividers → {Center}, Spacings → {1, 1}]]
```
Out[∘]=

vel2	ncars1
5	70
10	58
20	44
30	35
40	29
50	25
60	22
70	19

In[∘]:=
```
lp2 = ListPlot[table2, AxesLabel → {"vel", "ncars1"},
            PlotLabel → Style["Easing Traffic", Blue, 16],
            PlotStyle → {Red, PointSize[0.015]},
   PlotRange → {{0, 100}, {0, 100}},
            AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[∘]:=
```
model2 = a Exp[-k t];
```

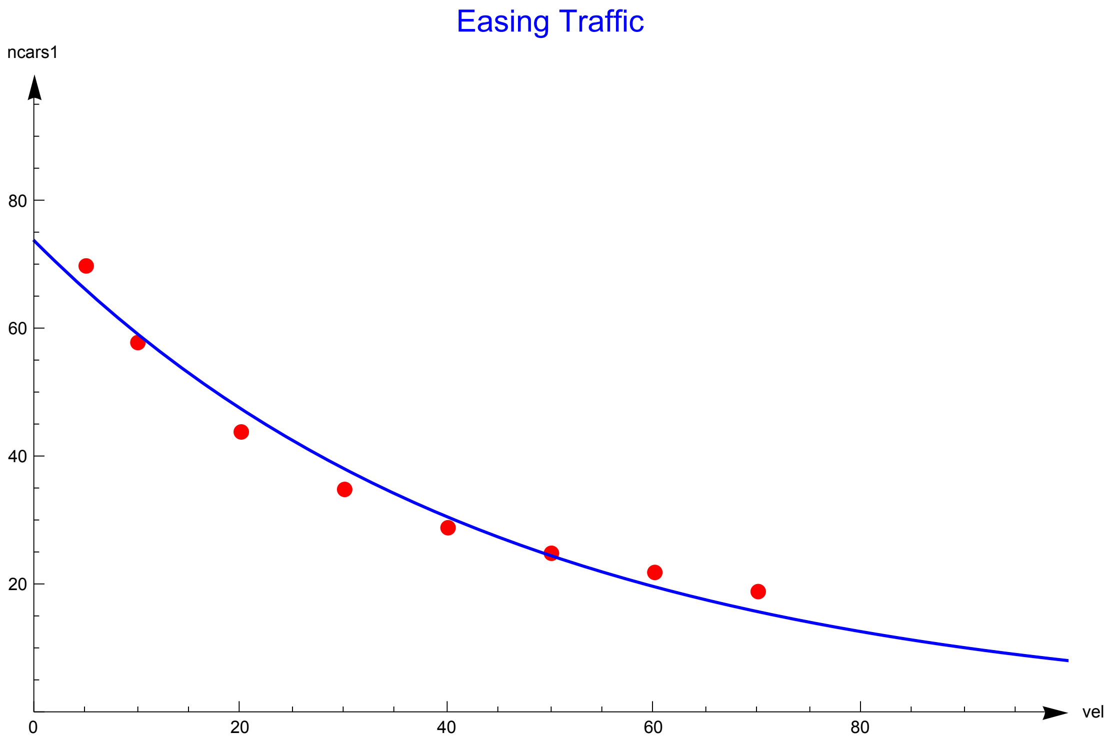
In[∘]:=
```
fit2 = FindFit[table2, model2, {a, k}, t]
```
Out[∘]=
```
{a → 73.8553, k → 0.0222245}
```

In[∘]:=
```
expfit[x_] := 73.9 * Exp[-0.022 x]
```

In[∘]:=
```
expfit[1]
```
Out[∘]=
```
72.292
```

In[∘]:=
```
plot2 = Plot[expfit[x], {x, 0, 100}, AxesLabel → {"ncars1", "dist"},
            PlotLabel → Style["Easing Traffic", Blue, 16],
            PlotStyle → {Blue, PointSize[0.015]},
   PlotRange → {{0, 100}, {0, 100}},
            AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[∘]:=
```
Show[lp2, plot2]
```

Out[∘]=


In[∘]:=
```
ncars2 = Floor[expfit[vel2]]
```

Out[∘]= {66, 59, 47, 38, 30, 24, 19, 15}

In[∘]:=
```
table3 = Transpose[{vel2, ncars2}]
```

Out[∘]= {{5, 66}, {10, 59}, {20, 47}, {30, 38}, {40, 30}, {50, 24}, {60, 19}, {70, 15}}

In[∘]:=
```
Text[Grid[Prepend[table3, {"vel2", "ncars2"}],
  Alignment → Center, Dividers → {Center}, Spacings → {1, 1}]]
```

Out[∘]=

vel2	ncars2
5	66
10	59
20	47
30	38
40	30
50	24
60	19
70	15

This model shows that the max flow is 88 car per min when the speed is 0 mph. This does not make sense.

Model 2

In[∘]:=
```
table4 = Transpose[{vel2, dist2, dist2 + 13}]
```
Out[∘]=
```
{{5, 6.25, 19.25}, {10, 15., 28.}, {20, 40., 53.}, {30, 75., 88.},
 {40, 120., 133.}, {50, 175., 188.}, {60, 240., 253.}, {70, 315., 328.}}
```

In[∘]:=
```
Text[Grid[Prepend[table4, {"vel2", "dist2", "dist2 + 13"}],
  Alignment → Center, Dividers → {Center}, Spacings → {2, 2, 2}]]
```
Out[∘]=

vel2	dist2	dist2 + 13
5	6.25	19.25
10	15.	28.
20	40.	53.
30	75.	88.
40	120.	133.
50	175.	188.
60	240.	253.
70	315.	328.

In[∘]:=
```
ncars3 = Floor[ncarsf[vel2, dist2 + 13]]
```
Out[∘]=
```
{22, 31, 33, 30, 26, 23, 20, 18}
```

In[∘]:=
```
table5 = Transpose[{vel2, dist2, dist2 + 13, ncars3}]
```
Out[∘]=
```
{{5, 6.25, 19.25, 22}, {10, 15., 28., 31}, {20, 40., 53., 33}, {30, 75., 88., 30},
 {40, 120., 133., 26}, {50, 175., 188., 23}, {60, 240., 253., 20}, {70, 315., 328., 18}}
```

In[∘]:=
```
table6 = Transpose[{vel2, ncars3}]
```
Out[∘]=
```
{{5, 22}, {10, 31}, {20, 33}, {30, 30}, {40, 26}, {50, 23}, {60, 20}, {70, 18}}
```

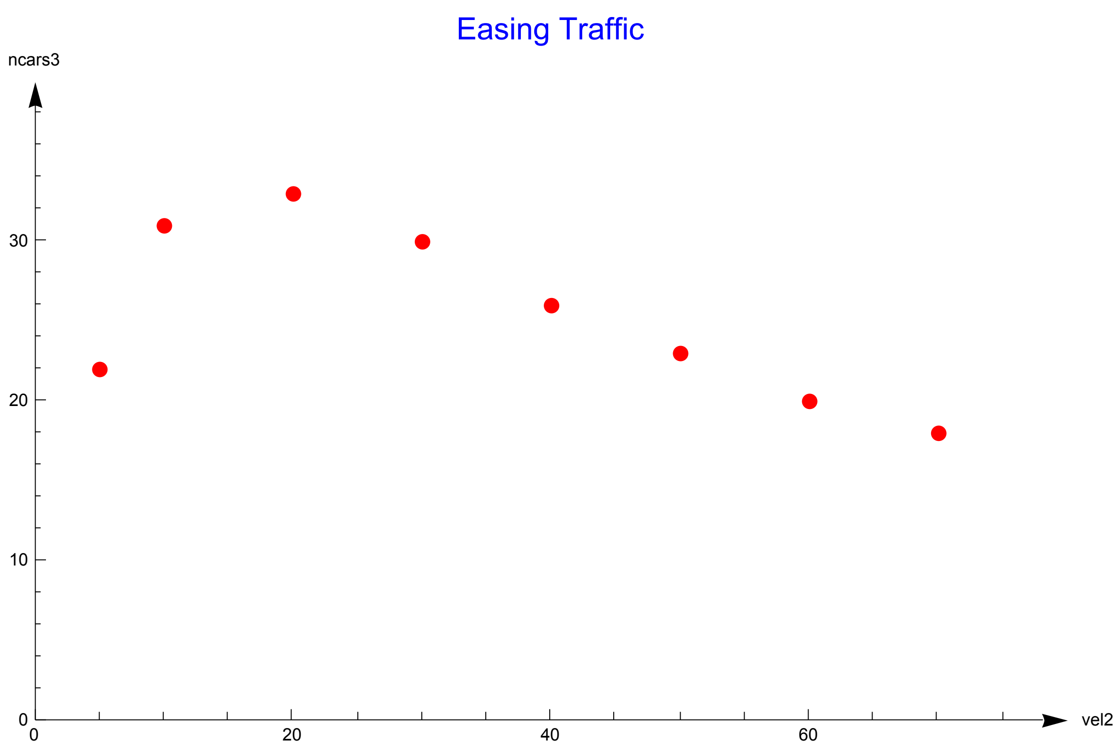
In[∘]:=
```
Text[Grid[Prepend[table5, {"vel2", "dist2", "dist2 + 13", "ncars3"}],
  Alignment → Center, Dividers → {Center}, Spacings → {2, 2, 2}]]
```

Out[∘]=

vel2	dist2	dist2 + 13	ncars3
5	6.25	19.25	22
10	15.	28.	31
20	40.	53.	33
30	75.	88.	30
40	120.	133.	26
50	175.	188.	23
60	240.	253.	20
70	315.	328.	18

In[∘]:=
```
lp3 = ListPlot[table6, AxesLabel → {"vel2", "ncars3"},
            PlotLabel → Style["Easing Traffic", Blue, 16],
            PlotStyle → {Red, PointSize[0.015]},
  PlotRange → {{0, 80}, {0, 40}},
            AxesStyle → Arrowheads[0.025], ImageSize → Large]
```

Out[∘]=



From the plot above, the max number of cars exiting the tunnel is approximately 34 cars per minute when the speed is about 15 miles per hour.

In[∘]:=
```
dist1[15]
```
Out[∘]=
```
26.25
```

For an expected speed of 15 mph, the expected separation between cars in the tunnel is 26 feet (approximately 2 car lengths).

Model 3

No thinking distance

In[∘]:=
```
ncar4 = Floor[ncarsf[vel2, vel2 + 13]]
```
Out[∘]=
```
{24, 38, 53, 61, 66, 69, 72, 74}
```

In[∘]:=
```
table7 = Transpose[{vel2, vel2, vel2 + 13, ncar4}]
```
Out[∘]=
```
{{5, 5, 18, 24}, {10, 10, 23, 38}, {20, 20, 33, 53}, {30, 30, 43, 61},
 {40, 40, 53, 66}, {50, 50, 63, 69}, {60, 60, 73, 72}, {70, 70, 83, 74}}
```

```
Text[Grid[Prepend[table7,
   {"vel2", "dist2 after thinking distace is 0", "dist2 + 13", "ncars4"}],
  Alignment → Center, Dividers → {Center}, Spacings → {2, 2, 2}]]
```

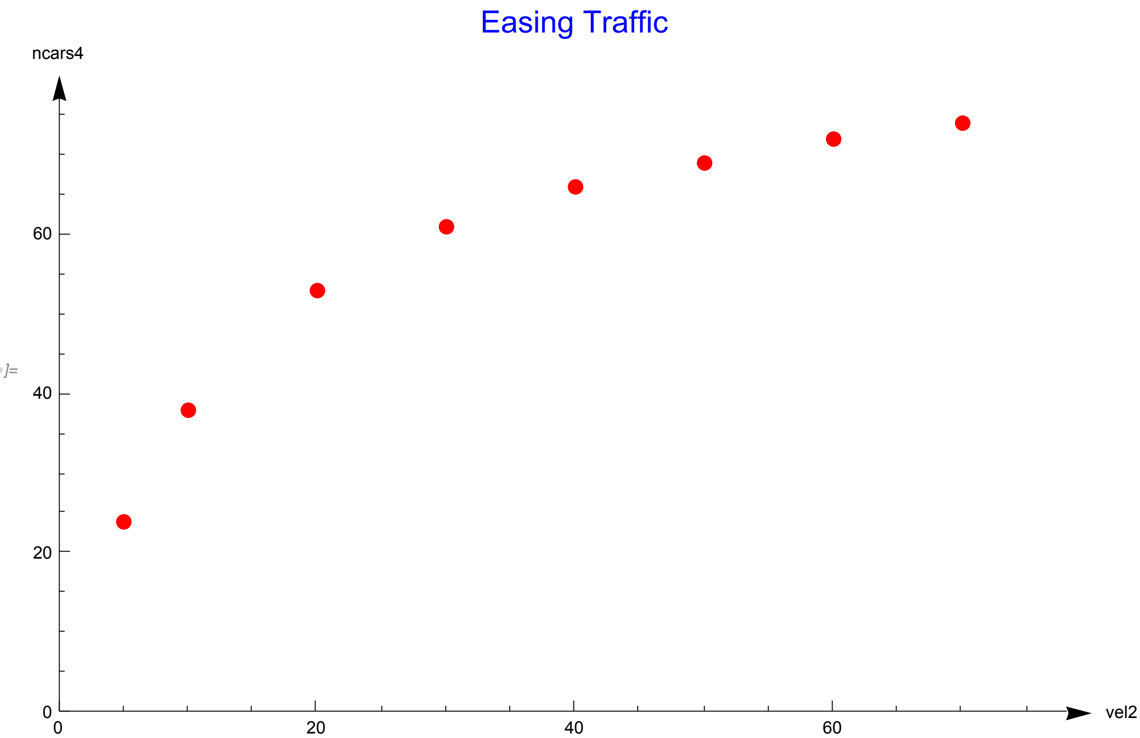
Out[∘]=

vel2	dist2 after separation is 0	dist2 + 13	ncars4
5	5	18	24
10	10	23	38
20	20	33	53
30	30	43	61
40	40	53	66
50	50	63	69
60	60	73	72
70	70	83	74

In[∘]:=
```
table8 = Transpose[{vel2, ncar4}]
```
Out[∘]=
```
{{5, 24}, {10, 38}, {20, 53}, {30, 61}, {40, 66}, {50, 69}, {60, 72}, {70, 74}}
```

In[∘]:=
```
lp3 = ListPlot[table8, AxesLabel → {"vel2", "ncars4"},
        PlotLabel → Style["Easing Traffic", Blue, 16],
        PlotStyle → {Red, PointSize[0.015]},
   PlotRange → {{0, 80}, {0, 80}},
        AxesStyle → Arrowheads[0.025], ImageSize → Large]
```

Out[∘]=



The maximum flow is when the speed is 70mph. The separation is 70 feet (about 5 car lengths). This is still highly unsafe, so model 4 is a compromise between models 2 and 3.

Model 4

In[∘]:=
```
sep[x_] = x + (x^2 / 40)
```

Out[∘]= $x + \frac{x^2}{40}$

```
separation = sep[vel2]
```

Out[∘]= {5, 12, 30, 52, 80, 112, 150, 192}

In[∘]:=
```
ncars5 = Floor[ncarsf[vel2, separation + 13]]
```

Out[∘]= {23, 34, 40, 40, 37, 35, 32, 29}

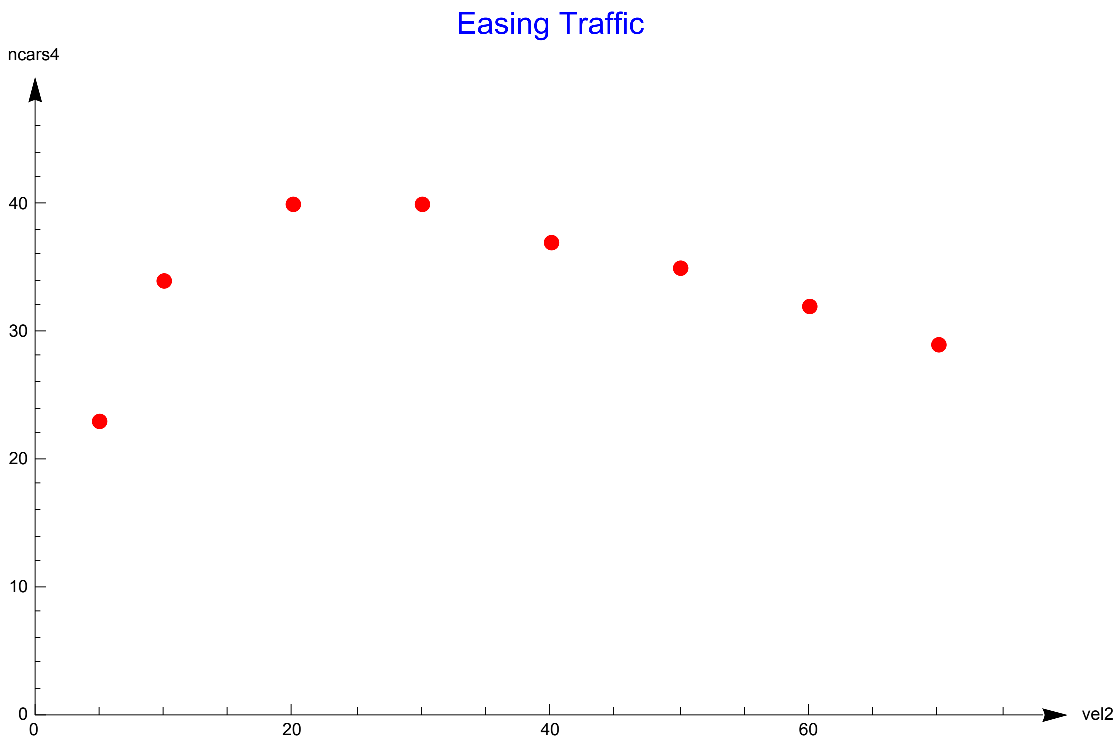
In[∘]:=
```
table9 = Transpose[{vel2, separation, separation + 13, ncars5}]
```

Out[∘]= {{5, 5, 18, 23}, {10, 12, 25, 34}, {20, 30, 43, 40}, {30, 52, 65, 40}, {40, 80, 93, 37}, {50, 112, 125, 35}, {60, 150, 163, 32}, {70, 192, 205, 29}}

In[∘]:=
```
table10 = Transpose[{vel2, ncars5}]
```
Out[∘]=
```
{{5, 23}, {10, 34}, {20, 40}, {30, 40}, {40, 37}, {50, 35}, {60, 32}, {70, 29}}
```

In[∘]:=
```
lp3 = ListPlot[table10, AxesLabel → {"vel2", "ncars4"},
              PlotLabel → Style["Easing Traffic", Blue, 16],
              PlotStyle → {Red, PointSize[0.015]},
  PlotRange → {{0, 80}, {0, 50}},
              AxesStyle → Arrowheads[0.025], ImageSize → Large]
```

Out[∘]=



Conclusion:

Based on the graph above, the maximum flow occurs when the speed limit is 20mph with separation of 4 cars. So, I would recommend a maximum speed of 20 MPH and a minimum separation of 4 cars during peak rush hour.